ECS455: Chapter 4

Multiple Access

4.4 m-sequence

How do we generate the spreading code c(t)?

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Binary Random Sequences

- While DSSS chip sequences must be generated *deterministically*, properties of binary random sequences are useful to gain insight into deterministic sequence design.
- A random binary chip sequences consists of i.i.d. bit values with probability one half for a one or a zero.
  - Also known as **Bernoulli sequences/trials**, "coin-flipping" sequences
- A random sequence of length N can be generated, for example, by flipping a fair coin N times and then setting the bit to a one for heads and a zero for tails.

$1-2x$

$H \to 1 \to -1$

$T \to 0 \to 1$

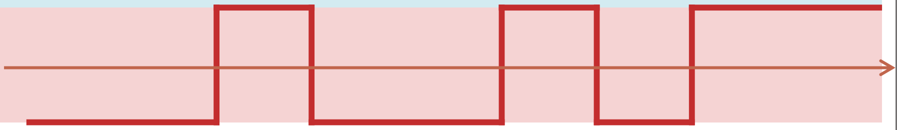
Binary Random Sequence

fair $p = 1/2 = 50\%$

Support $\{H, T\}$ $\{0, 1\}$ $\{\pm 1\}$

	X_{-4}	X_{-3}	X_{-2}	X_{-1}	X_{-0}	X_1	X_2	X_3	X_4
Coin-flipping sequence	H	H	T	H	H	T	H	T	T
Bernoulli trials/sequence	1	1	0	1	1	0	1	0	0
Binary (indp.) random sequence	-1	-1	1	-1	-1	1	-1	1	1

Waveform



- These names are simply many versions of the same sequence/process.
- You should be able to convert one version to others easily.
- Some properties are conveniently explained when the sequence is expressed in a particular version.

Properties of Binary Random Sequences:

- Consider the sequence $X_1, X_2, X_3, \ldots, X_n, \ldots$
- Disadvantages
  - Can not further "compress" the sequence
  - Difficult to convey the sequence from the Tx to Rx
  - Require large storage at both Tx and Rx
- Advantages
  - Random = unpredictable
  1. Balanced property
  2. Run length property
  3. Shift property

Properties of Binary Random Sequences: Balanced Property

- $\{0,1\}$ version

The proportion of 1s in the sequence =

$$\frac{1}{N}\sum_{i=1}^{N} X_i \xrightarrow[\text{LLN}]{N\to\infty} \mathbb{E}[X_i] = 0\times\frac{1}{2}+1\times\frac{1}{2}=\frac{1}{2}$$

- $\{\pm 1\}$ version

$$\frac{1}{N}\sum_{i=1}^{N} X_i \xrightarrow[\text{LLN}]{N\to\infty} \mathbb{E}[X_i] = (-1)\times\frac{1}{2}+1\times\frac{1}{2}=0$$

Runs: An Example

For us, there are only two types
- consecutive 1s
- consecutive 0s

- A run is a subsequence of consecutive identical symbols within the sequence.
- The following sequence contains 16 runs

0001111100110100100001010111011

$\underbrace{000}$ A run of 0s (length = 3)

$\underbrace{11111}$ A run of 1s (length = 5)

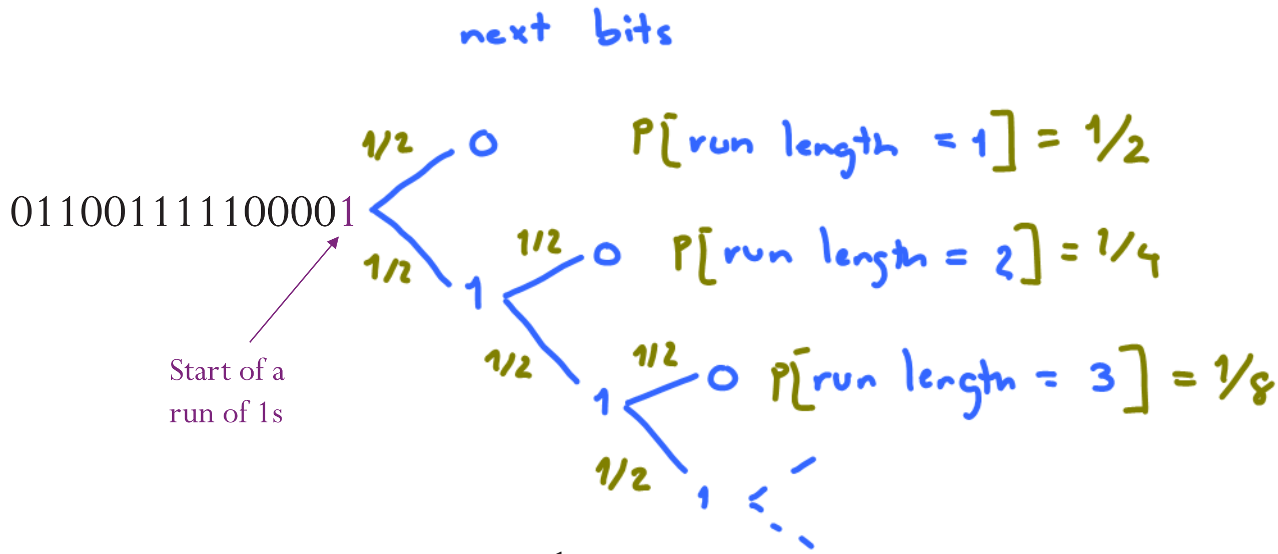
- Rel. Freq of Run Lengths

Run Length	Rel. Freq.
5	1/16
4	1/16
3	2/16
2	4/16
1	8/16

- Rel. Freq of Runs

11111	1/16
0000	1/16
111	1/16
000	1/16
11	2/16
00	2/16
1	4/16
0	4/16

Properties of Binary Random Sequences: Run Length Property



$P[\text{run length} = \ell] = \dfrac{1}{2^{\ell}}$ ➡ As ℓ increases, P is reduced.

➡ Small probability of having log runs.

FYI: Run-Length Encoding (RLE)

- A very simple form of lossless data compression in which runs of data (that is, sequences in which the same data value occurs in many consecutive data elements) are stored as a single data value and count, rather than as the original run.
- Most useful on data that contains many such runs.
- Example: Consider a screen containing plain black text on a solid white background.
  A line, with B representing a black pixel and W representing white, might read as follows:

  WWWWWWWWWWWWBWWWWWWWWWWWWBBBWWWWWWWWWWWWWWWWWWWWWWWWBWWWWWWWWWWWWWW

  With a RLE data compression algorithm applied to the above line, it can be rendered as follows:

  12W1B12W3B24W1B14W

Properties of Binary Random Sequences: Shift Property

Original sequence: $X_1 \quad X_2 \quad X_3 \quad X_4 \quad \cdots \quad X_N$

Shifted sequence: $X_{-1} \quad X_0 \quad X_1 \quad X_2 \quad \cdots \quad X_{N-2}$

(amount of shift (delay) = 2)

- When the shifted amount = 0, the two sequences are exactly the same.
- When the shifted amount = s, we want to compare X_j and X_{j-s}.
  - What proportion are the same? ½
  - What proportion are different? ½
- Recall that the numbers in the sequence are independent results (from several Bernoulli trials)

X_j	X_{j-s}	Probability
0	0	¼
0	1	¼
1	0	¼
1	1	¼

Properties of Binary Random Sequences: Shift Property

- $\{0,1\}$ version: The comparison is done via the XOR ($\oplus$) operation
  - $x \oplus y = 0$ iff they are the same
  - $x \oplus y = 1$ iff they are different
- $\{\pm 1\}$ version: The comparison is done via the multiplication operation
  - $x \times y = 1$ iff they are the same
  - $x \times y = -1$ iff they are different

Key randomness properties

[Golomb, 1967][Viterbi, 1995, p. 12] Binary random sequences with length N asymptotically large have a number of the properties desired in spreading codes

- **Balanced property**: Equal number of ones and zeros.
  - Should have no DC component to avoid a spectral spike at DC or biasing the noise in despreading
- **Run length property**: The run length is generally short.
  - half of all runs are of length 1
  - a fraction $1/2^n$ of all runs are of length n (Geometric)
  - Long runs reduce the BW spreading and its advantages
- **Shift property**: If they are shifted by any nonzero number of elements, the resulting sequence will have half its elements the same as in the original sequence, and half its elements different from the original sequence.

[Goldsmith, 2005, p. 387 & Viterbi, p. 12]

Pseudorandom Sequence

- A deterministic sequence that has the balanced, run length, and shift properties as it grows *asymptotically large* is referred to as a **pseudorandom sequence** (noiselike or pseudonoise (PN) signal).
- Ideally, one would prefer a random binary sequence as the spreading sequence.
- However, practical synchronization requirements in the receiver force one to use **periodic** Pseudorandom binary sequences.

- m-sequences
- Gold codes
- Kasami sequences
- Quaternary sequences
- Walsh functions